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Possible resolution of black hole singularities from large N gauge theory

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Abstract: We point out that the recent conjecture relating large N gauge theories to string theory in anti-de Sitter spaces offers a resolution in principle of many problems in black hole physics. This is because the gauge theory also describes spacetimes which are not anti-de Sitter, and include black hole horizons and curvature singularities.

Keywords: Black Holes in String Theory, Black Holes.

JHEP04(1998)015

Contents

1. Introduction 1

2. Near-extreme D-three branes 3

3. Discussion 5

1. Introduction

A longstanding goal of quantum gravity is to smooth out, or resolve, the singularities of classical general relativity. It is now well known that many spacetimes which are singular from the standpoint of general relativity are in fact nonsingular when viewed as solutions to string theory. An early example of this was orbifold singularities, in which the curvature remains bounded near the singular point. This has since been extended to more serious curvature singularities such as the conifold singularity [1]. However, the singularities inside a black hole have not yet been resolved in the context of string theory (although some interesting partial results have been obtained [2]).

A related issue concerns the description of spacetime inside the event horizon. In the original calculations of the entropy of extreme and near-extreme black holes in string theory [3, 4, 5], one considered a configuration of D-branes and counted the number of states of given energy at weak coupling. (These excitations are described by a supersymmetric gauge theory.) One then extrapolated to strong coupling and compared this number with the area of the event horizon of the corresponding black hole. Since the area of the event horizon is much less than the string scale at weak coupling, it wasn't clear what the relation was between the quantum states of the gauge theory and the region of spacetime behind the horizon. There was speculation that perhaps the branes were located near the horizon, replacing the region inside [4], or that the description of the gauge theory interacting with the bulk supergravity modes was appropriate only for outside observers. Another description would then be needed for infalling observers.

Recent developments in string theory have shed light on both of these issues. It was proposed in [6] that for large N and strong coupling ($g_{YM}^2 N \gg 1$), supersymmetric $U(N)$ gauge theories are equivalent to string theory in particular supergravity backgrounds. (For earlier work along these lines, see [7, 8].) The basic idea is to consider N D-branes in string theory in a low energy limit. From an effective field theory analysis, one argues that the degrees of freedom on the branes decouple from the bulk modes in this limit. Starting with the supergravity solution describing the branes and applying the same limit, one obtains a region of spacetime near the horizon. Since one starts with the same theory and applies the same limiting procedure, the resulting descriptions should be equivalent. This bold conjecture has been clarified in [9, 10] and investigated in a number of recent papers [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28], with promising results. In this paper, we will see that if we follow the argument of [6] for near-extreme branes, we find that the gauge theory at finite energy density is equivalent to string theory in an asymptotically anti-de Sitter black hole solution, including the region behind the horizon.

Let us first review the argument in the extreme case. To be concrete, we consider N D-three branes in type IIB string theory. The supergravity solution has constant dilaton and metric

$$ds^2 = f^{-1/2}(-dt^2 + dy_i dy^i) + f^{1/2}(dr^2 + r^2 d\Omega_5)\,, \tag{1.1}$$

with

$$f(r) = 1 + \frac{4\pi g N \alpha'^2}{r^4}\,, \tag{1.2}$$

where g is the string coupling ($g \sim g_{YM}^2$), and $y_i,\ i = 1, 2, 3$ denote the directions along the branes. The horizon is at $r = 0$, and these coordinates (with $r > 0$) cover only the region outside the horizon. The maximal analytic extension contains an infinite number of asymptotically flat regions, but no singularities [29]. The low energy limit consists of setting $r = U\alpha'$, and taking $\alpha' \to 0$ with U and g fixed. This effectively restricts attention to the region near the horizon. The resulting metric is

$$ds^2 = \frac{U^2}{\sqrt{4\pi g N}}(-dt^2 + dy_i dy^i) + \frac{\sqrt{4\pi g N}}{U^2} dU^2 + \sqrt{4\pi g N} d\Omega_5 \tag{1.3}$$

in string units. This metric describes the product of S^5 and five dimensional anti-de Sitter space (AdS_5), both with radii $R^2 = \sqrt{4\pi g N}$. The gauge theory associated with N D-three branes is four dimensional $U(N)$ $\mathcal{N} = 4$ super Yang-Mills. The conjecture is that the strong coupling limit of this gauge theory is equivalent to string theory on $AdS_5 \times S^5$, with the above radii.

In [12] it was argued that the gauge theory actually describes string theory on both sides of the horizon $U = 0$. This is because the gauge theory has a conformal symmetry group $SO(2,4)$, which is supposed to be identified with the isometry group of the near horizon geometry. $SO(2,4)$ is indeed the symmetry group of AdS_5, since AdS_5 is the surface $-UV + \eta_{\mu\nu}X^\mu X^\nu = -R^2$ in a flat spacetime with signature $(2,4)$, $\mathbf{R}^{(2,4)}$. However, the region outside the horizon ($U > 0$) is not invariant under this full group, but only under a subgroup which leaves a null direction in $\mathbf{R}^{(2,4)}$ invariant. So the gauge theory, which is invariant under the full group, must describe string theory on both sides of the horizon. Furthermore, to properly realize the conformal symmetry, the gauge theory should not be defined on $\mathbf{R}^4$ but rather on $S^3 \times \mathbf{R}$. Time evolution on $S^3 \times \mathbf{R}$ corresponds in AdS_5 to evolution with respect to a global timelike symmetry which does not leave the region $U > 0$ invariant.

2. Near-extreme D-three branes

The argument of [12] is encouraging, but does not address the issue of black hole singularities, since the extremal three brane is regular everywhere. The horizons in the AdS_5 geometry are also not true event horizons, since they depend on a particular choice of Killing field. For the near-extremal three brane, the metric is given by

$$ds^2 = f^{-1/2}(-hdt^2 + dy_i dy^i) + f^{1/2}(h^{-1}dr^2 + r^2 d\Omega_5)\,, \tag{2.1}$$

where f is given by (1.2) and

$$h(r) = 1 - \frac{{r_0}^4}{r^4}\,. \tag{2.2}$$

The global structure of this solution is similar to the Schwarzschild solution, and is shown in fig. 1. This nonextremal solution corresponds to excited D-branes whose dynamics are now described by the gauge theory at a finite energy density.

One might have expected that the limit which decouples the gauge theory from the bulk would restrict to the region near the horizon, and both the singularity and the asymptotically flat region would be removed. This is not the case. In fact one obtains the metric

$$\begin{aligned} ds^2 = & \frac{U^2}{\sqrt{4\pi g N}}\left[-\left(1-\frac{U_0^4}{U^4}\right)dt^2 + dy_i dy^i\right] \\ & + \frac{\sqrt{4\pi g N}}{U^2}\left(1-\frac{U_0^4}{U^4}\right)^{-1} dU^2 + \sqrt{4\pi g N}\, d\Omega_5\,. \end{aligned} \tag{2.3}$$

Note that the five sphere decouples throughout the spacetime (2.3). The Penrose diagram for the remaining five dimensional spacetime is shown in fig. 2. This is similar

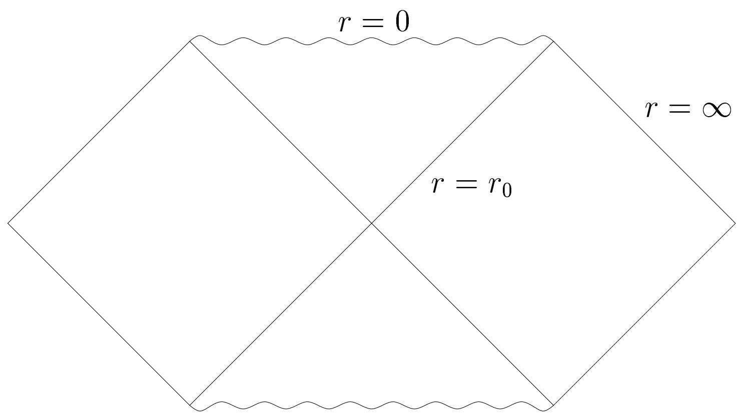

Figure 1: Causal structure for the near-extreme 3-brane. Each point represents $S^5 \times R^3$.

to the Schwarzschild anti-de Sitter metric, but the symmetries are different; there are translational invariances along the y^i directions rather than the spherical symmetry of Schwarzschild anti-de Sitter. The five dimensional spacetime is asymptotically AdS_5, but it is not locally AdS_5. While the Ricci tensor is constant $R_{\mu\nu} = -4(4\pi gN)^{-1/2} g_{\mu\nu}$, the Weyl tensor is nonzero, and given by

$$C_{ijij} \sim \frac{U_0^4}{U^4\sqrt{4\pi gN}}\,, \tag{2.4}$$

where the indices i, j denote components with respect to an orthonormal basis in the five-dimensional metric, and there is no sum on i, j. The curvature clearly diverges as $U \to 0$. The five dimensional spacetime in (2.3) is a solution to Einstein's equation with a negative cosmological constant, and is analogous to four-dimensional metrics found earlier [30].

Since the near extremal solution just corresponds to the gauge theory at finite energy density ($\rho \sim U_0^4$), its clear that the gauge theory can describe string theory in backgrounds other than anti de Sitter space. Furthermore, since the entire spacetime shown in fig. 2 arises in the scaling limit, the gauge theory must describe string theory on this entire space.

One might worry that the gauge theory only captures the physics outside the horizon, since the natural time on the worldbrane (for $\mathbf{R}^4$ topology) only covers the external region. However the same argument could have been made in the extremal case, and we have already seen that the gauge theory naturally includes the region beyond the horizon in this case. It is true that the solution (2.3) has less symmetry than AdS_5, so there is no direct analog of the symmetry argument of [12] to guarantee that the region inside the horizon is included. However, the gauge theory is the same as in the extreme case; one is simply considering different states. Also, the difference between S^3 and $\mathbf{R}^3$ should not matter for reasonable energy densities.

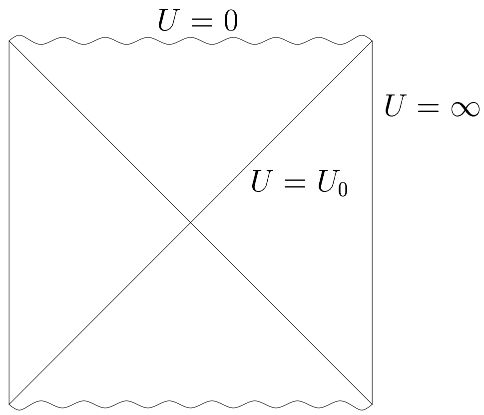

Figure 2: Causal structure for the decoupled region.

Note that if we considered a Euclidean version of the solution, as in [10], we would find that the Euclidean section arises from analytically continuing the region outside the horizon, and we would expect the Euclidean version of the gauge theory to correspond to string theory on this Euclidean manifold. This does not contradict our Lorentzian argument. String theory is consistent on the Euclidean solution since it is a complete space. String theory cannot be consistently defined on the region outside the event horizon of the Lorentzian solution, since strings will fall in.

The supergravity approximation will break down once the curvature becomes large, or if there are compact directions smaller than the string scale (so winding modes

JHEP04(1998)015

become light). For large gN, the curvature is small near the horizon and everywhere outside. If one periodically identifies the y_i, the radii of these circles shrink as one decreases U. At small enough U, the circles will be smaller than the string scale, and it would be more appropriate to study the T-dual metric. However, since one can compactify the y_i with arbitrary periods L_i, for a given energy density, one can arrange for the circles to stay larger than the string scale until the local curvature becomes large. Then the supergravity approximation breaks down only near the singularity in the five-dimensional spacetime. Since the gauge theory is perfectly nonsingular, it must provide a regular description of the spacetime, including the singular region.

The near extremal three brane will Hawking radiate. Since the finite temperature gauge theory is clearly in equilibrium, it cannot describe a time-dependent process such as the evaporation of the three brane. Thus, the finite temperature gauge theory must describe the brane in thermal equilibrium with a gas of string modes in anti-de Sitter space. The negative cosmological constant acts like a confining box, so unlike the asymptotically flat case, the thermal gas will have finite energy.

3. Discussion

We have argued that the analysis of [6] for near-extreme three branes implies that the finite temperature large N gauge theory is equivalent to string theory on a background spacetime which describes a black hole in anti-de Sitter space. In particular, the gauge theory describes the region inside the black hole's horizon, and, since it is perfectly regular, provides a resolution of the singularity inside the black hole. Similar results apply for the near-extremal versions of the other cases discussed in [6]. For the two brane and five brane in M theory, the discussion in the previous section is carried over essentially unchanged. For the near-extremal black string obtained from a D1+D5 system in six dimensions (and for five-dimensional black strings), the black hole solution is just the BTZ black hole [31]. The geometry is locally AdS_3 in this case, and there is no curvature singularity. The near-extremal six dimensional black string does have a curvature singularity, and it also has two horizons: an event horizon and inner horizon. The decoupling limit includes both horizons, but not the singularity.

If we take the y_i to be periodically identified in (2.3), the spacetime has two asymptotic boundaries, corresponding to the two sides of the Penrose diagram in fig. 2. It seems likely that the interpretation of this is the same as in general relativity: the second asymptotic region is probably unphysical, and does not arise when the process of formation is included. For example, suppose one starts with N D three branes spread out over a ball in the noncompact directions. This should act like a $q = m$ charged dust. If the branes are given some initial inward radial velocity, they will collapse to form the nonextreme black hole, but the Penrose diagram will now have only one asymptically AdS region.

Work is underway to find further support for this remarkable connection between gauge theory and black hole singularities. One issue that evidently deserves further study is understanding the causal structure of the spacetime from the gauge theory point of view. That is, we would like to ask why a test three brane, say, is restricted to move within the light cones of the metric (2.3). At weak coupling, the causal dynamics of branes follows from the Dirac-Born-Infeld action. Here we have only the super Yang-Mills action, but its possible that the DBI action arises as a low energy effective action for probe branes. Studying infalling branes (as in [32]) seems a sensible next step to obtaining a greater understanding of the connection between gauge theory and geometry.

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References

[1] A. Strominger, *Massless black holes and conifolds in string theory*, *Nucl. Phys.* **B 451** (1995) 96 [hep-th/9504090].

[2] E. Martinec, *String field theory in curved space-time and the resolution of spacelike singularities*, *Class. and Quant. Grav.* **13** (1996) 63 [hep-th/9509149].

[3] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, *Phys. Lett.* **B 379** (1996) 99 [hep-th/9601029].

[4] C.G. Callan, Jr. and J.M. Maldacena, *D-brane approach to black hole quantum mechanics*, *Nucl. Phys.* **B 472** (1996) 591 [hep-th/9602043].

[5] G.T. Horowitz and A. Strominger, *Counting states of near extremal black holes*, *Phys. Rev. Lett.* **77** (1996) 2368 [hep-th/9602051].

[6] J. Maldacena, *The large N limit of superconformal field theories and supergravity*, hep-th/9711200.

[7] I.R. Klebanov, *World volume approach to absorption by nondilatonic branes*, *Nucl. Phys.* **B 496** (1997) 231 [hep-th/9702076].

[8] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, *String theory and classical absorption by three-branes*, *Nucl. Phys.* **B 499** (1997) 217 [hep-th/9703040].

[9] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from noncritical string theory*, hep-th/9802109.

[10] E. Witten, *Anti-de Sitter space and holography*, hep-th/9802150.

[11] N. Itzhaki, J. Maldacena, J. Sonnenschein and S. Yankielowicz, *Supergravity and the large N limit of theories with sixteen supercharges*, hep-th/9802042.

[12] G.T. Horowitz and H. Ooguri, *Spectrum of large N gauge theory from supergravity*, hep-th/9802116.

[13] S. Kachru and E. Silverstein, *4-D conformal theories and strings on orbifolds*, hep-th/9802183.

[14] R. Kallosh, J. Kumar, and A. Rajaraman, *Special conformal symmetry of world volume actions*, hep-th/9712073.

[15] M. Berkooz, *A supergravity dual of a (1,0) field theory in six-dimensions*, hep-th/9802195.

[16] S.-J. Rey and J. Yee, *Macroscopic strings as heavy quarks in large N gauge theory and anti-de Sitter supergravity*, hep-th/9803001.

[17] J. Maldacena, *Wilson loops in large N field theories*, hep-th/9803002.

[18] A. Lawrence, N. Nekrasov and C. Vafa, *On conformal field theories in four-dimensions*, hep-th/9803015.

[19] S.S. Gubser, A. Hashimoto, I.R. Klebanov and M. Krasnitz, *Scalar absorption and the breaking of the world volume conformal invariance*, hep-th/9803023.

[20] L. Castellani *et al.*, *G/H M-branes and $AdS_{(p+2)}$ geometries*, hep-th/9803039.

[21] J. Gomis, D. Mateos, J. Simon and P.K. Townsend, *Brane intersection dynamics from branes in brane backgrounds*, hep-th/9803040.

[22] P. Claus, R. Kallosh, J. Kumar, P. Townsend and A.V. Proeyen, *Conformal theory of M2, D3, M5 and D1-branes + D5-branes*, hep-th/9801206.

[23] S. Ferrara and C. Fronsdal, *Conformal Maxwell theory as a singleton field theory on $AdS(5)$, IIB three-branes and duality*, hep-th/9712239.

[24] S. Ferrara and C. Fronsdal, *Gauge fields as composite boundary excitations*, hep-th/9802126.

[25] S. Ferrara, C. Fronsdal and A. Zaffaroni, *On $N = 8$ supergravity on $AdS(5)$ and $N = 4$ superconformal Yang-Mills theory*, hep-th/9802203.

[26] M. Flato and C. Fronsdal, *Interacting singletons*, hep-th/9803013.

[27] O. Aharony, Y. Oz and Z. Yin, *M theory on $AdS_p \times S^{11-p}$ and superconformal field theories*, hep-th/9803051.

[28] S. Minwalla, *Particles on $AdS_{4/7}$ and primary operators on $M_{2/5}$ brane worldvolumes*, hep-th/9803053.

[29] G.W. Gibbons, G.T. Horowitz and P.K. Townsend, *Higher dimensional resolution of dilatonic black hole singularities*, *Class. and Quant. Grav.* **12** (1995) 297 [hep-th/9410073].

[30] R. B. Mann, *Pair production of topological anti-de Sitter black holes*, *Class. and Quant. Grav.* **14** (1997) L109 [gr-qc/9607071].

[31] M. Banados, C. Teitelboim and J. Zanelli, *The black hole in three-dimensional spacetime*, *Phys. Rev. Lett.* **69** (1992) 1849 [hep-th/9204099].

[32] J. Maldacena, *Probing near extremal black holes with D-branes*, *Phys. Rev.* **D 57** (1998) 3736 [hep-th/9705053].